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Some Remarks on Nonlocal Field Theory and Space-Time Quantization

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Abstract

From the point of view that the charge and mass of an electron is of dynamical origin and quantization of charge in units of e is related to the space-time quantization as developed in an earlier paper, we here show that it is possible to consider that the internal space within the elementary domain of the quantized space-time world is not governed by Lorentz invariance. This helps us to develop a consistent theory of nonlocal fields for extended particles where the infinite mass degeneracy is avoided. Moreover, this ensures the convergence of nonlocal field theories and suggests that massless particles like photons and neutrinos, though they may be taken to be of extended structure, will appear only as point particles in the physical world. In this picture, Lorentz invariance appears to be a consequence of the distribution of matter and energy in the Universe, and this may be taken to be another interpretation of Mach's principle.

1. *Introduction*

In a recent paper (Bandyopadhyay, 1973a—hereafter referred to as I) we have argued that the quantization of charge in units of e can be taken to be a consequence of the space-time quantization when charge is considered to be of dynamical origin. In fact, we have shown in I that the charge and mass of an electron (as well as of a muon) can be taken to occur as a result of n photon–neutrino weak interactions, when photons and neutrinos are represented as nonlocal fields and n is given by the relation $e \approx ng$, g being the photon–neutrino weak coupling constant ($g \simeq 10^{-10}\ e$) (Bandyopadhyay, 1968). In this model electron and muon are depicted as $(\nu_e S)$ and $(\nu_\mu S)$, respectively, where S represents the system of photons interacting weakly at n space-time points with the extended structure of a two-component neutrino. The two other components corresponding to the positive and negative energy states are formed when the form factor associated with the interaction changes its sign, implying that particles and antiparticles are mirror reflections of each other (Bandyopadhyay, 1973b). This procedure helps us to unify weak and electromagnetic interactions and the accompanied violation of symmetry generates the photon as a Goldstone boson (Bandyopadhyay, 1973a). If we take that the charge of a hadron is also due to the presence of a lepton in its structure (Bandyopadhyay, 1975), then the charge spectrum of all hadrons can be interpreted on the basis of this concept of dynamical origin of charge.

In this picture, it is possible to show that the quantization of charge in units of e is related to the quantization of space-time, where each quantized space-time domain is determined by the region accommodating the specific n number of weak interactions involving extended structures of n photons and one neutrino, also considered to be of extended structure. However, as we know, it is not yet properly understood how massless particles with extended structures can be described. Moreover, as Yukawa has pointed out, there are difficulties due to mass degeneracy in the simple model of non-local fields (Yukawa, 1965, 1973). In this context it is worthwhile to mention that a model of leptons has been constructed out of n photons weakly interacting with the neutrino in the frame work of nonlocal field theory (Bandyopadhyay, 1973a). This model suggests a preferential direction in the space within the fundamental domain, i.e., the internal space should be such as to violate the Lorentz invariance. Taking this into consideration we shall here show that the nonlocal fields representing the extended particles can be described in a consistent way.

2. *Nonlocal Field Theory and the Infinite Degeneracy of Mass States*

The concept of nonlocal field was first introduced by Yukawa (1950) to incorporate new degrees of freedom that might help us to understand the internal quantum numbers of hadrons. However, the goal could not be successfully achieved; besides, certain inconsistencies like infinite degeneracy of mass states appear in the simple model of nonlocal fields. In fact, in the description of nonlocal fields it is generally considered that the field is a function of two points and it can be represented by a matrix $\langle X'_\mu | \psi | X''_\mu \rangle$. The principle of reciprocity is understood as a symmetry of natural laws with respect to the commutators $[p_\mu, \psi]$ and $[x_\mu, \psi]$. Alternatively, ψ can be regarded as a function $\psi(x_\mu, r_\mu)$ of external coordinates $X_\mu = (x'_\mu + x''_\mu)/2$ and internal coordinates $r_\mu = x'_\mu - x''_\mu$. For a scalar field, the free field equation can be written as

$$F\left(\frac{\partial}{\partial r_\mu}, r_\mu, \frac{\partial}{\partial r_\mu}\right) \phi(X_\mu, r_\mu) = 0 \tag{2.1}$$

Assuming that F can be factorized in terms of a d'Alembertian operator for the external coordinates and an operator $F(r)$ depending only on the internal coordinates, we can write

$$\left[-\frac{\partial^2}{\partial X_\mu \partial X_\mu} + F^{(r)}\left(r_\mu r_\mu, \frac{\partial^2}{\partial r_\mu \partial r_\mu}, r_\mu \frac{\partial}{\partial r_\mu}\right)\right] \phi = 0 \tag{2.2}$$

In this case, ϕ can be solved in the product form

$$\phi = U(X)X(r) \tag{2.3}$$

where $U(X)$ and $X(r)$ must satisfy the equations

$$\left(\frac{\partial^2}{\partial X_\mu \partial X_\mu} - \mu\right) U(X) = (F^{(r)} - \mu)X(r) = 0 \tag{2.4}$$

Yukawa (1965, 1973) has considered the case of the harmonic oscillator. In the case of the four-dimensional oscillator model, the operator F is of the form

$$F = -\frac{\partial^2}{\partial X_\mu \partial X_\mu} + \tfrac{1}{2}\left(-\frac{\partial^2}{\partial r_\mu \partial r_\mu} + \frac{1}{\lambda 4} r_\mu r_\mu\right) \tag{2.5}$$

Now it is noted that for the solution of the equation

$$(F^{(r)} - \mu)X(r) = 0 \tag{2.6}$$

the eigenvalues μ are not positive definite. If we assign to each mode of vibration four quantum numbers n_1 in the direction of X, n_2 in the direction of Y, n_3 in the direction of Z, and another vibrational quantum number n_0 in the time direction, then the eigenvalue of $F^{(r)}$ is proportional to $n_1 + n_2 + n_3 - n_0$. Obviously, this leads to the degeneracy of the eigenvalues. For example, if we take the case $\mu = 0$, then there are an infinite number of different combinations of n_1, n_2, n_3, and n_0. Thus, if we accept this formalism, there is an infinite number of different types of particles, all of them having the same mass.

Yukawa (1965, 1973) has pointed out that there may be two ways to obviate these difficulties: (i) to consider a nonunitary representation of the Lorentz group, and (ii) to introduce the coupling between external and internal motions. Though by adoption of (ii) the difficulties of infinite degeneracy can be removed, the picture becomes far from simple. However, in this context Pais (1953) emphatically remarked that there is no a priori reason at all why the internal space should be governed by the Lorentz group, and if we demand that the internal space is not governed by the Lorentz group, then the difficulties related to the infinite mass degeneracy problem are removed. But the idea that the internal space does not obey this Lorentz symmetry is not favored for two reasons. Firstly, it was Yukawa's intention to correlate the internal quantum numbers of hadrons like isospin, strangeness and baryon number with the degrees of freedom connected with the internal space. Secondly, if we transform Yukawa's nonlocal field theory to the local field theory with nonlocal interaction, the form factor of the latter theory becomes connected with the variables of the internal space. So, if the internal space variables do not obey Lorentz symmetry, the form factors of the nonlocal interaction theory will have no relativistic invariance.

Here we point out that the description of electron (and muon) on the basis of the dynamical origin of charge and mass and the necessary requirement of space-time quantization can indeed accommodate the idea that the internal space within the quantized domain is not governed by Lorentz symmetry (Bandyopadhyay, 1974). According to this idea of space-time quantization, each quantized domain becomes the seat of an electron (as well as muon) and within this domain no measurement is possible. When the domain is filled up by the n number of photons interacting at different space-time points with the extended structure of a neutrino $\nu_e(\nu_\mu)$, a massive and charged particle like an electron (muon) is formed and we get into the world of Lorentz symmetry. For details, let us recapitulate the previous calculations as presented in I.

Let us consider the two-component spinor wave function $\psi(X, r)$, where X and r are external and internal space-time variables. It is considered that $\psi(X, r)$ satisfies the relation

$$\psi(x) = \int d^4r\psi(X, r) \tag{2.7}$$

It is further contended that the nonlocal spinor $\psi(X, r)$ obeys the Dirac equation in terms of the variable X

$$\left(\gamma^\mu \frac{\partial}{\partial X_\mu} + M\right) \psi(X, r) = 0 \tag{2.8}$$

The Spinor current is expressed as

$$C^\mu(x) = \int d^4r\, d^4S\bar{\psi}(X, r)\gamma_\mu\psi(X, s) \tag{2.9}$$

Now assuming that the electromagnetic field quantity $A_\mu(Y, t)$ also satisfies a similar relation

$$A_\mu(Y) = \int d^4tA_\mu(X, t) \tag{2.10}$$

we take n photon fields at different space-time points in the external space as follows:

$$A_\mu(Y - \tfrac{1}{2}\epsilon_1) + A_\mu(Y + \tfrac{1}{2}\epsilon_1) + A_\mu(Y - \tfrac{1}{2}\epsilon_2) + A_\mu(Y + \tfrac{1}{2}\epsilon_2) + \cdots + A_\mu(Y - \tfrac{1}{2}\epsilon_m) + A_\mu(Y + \tfrac{1}{2}\epsilon_m) \tag{2.11}$$

From this, we see that when $\epsilon \to 0$, the expression just reduces to the single point potential given by $nA_\mu(Y)$ when $n = 2m$. Thus the interaction Lagrangian for n photon weak interactions with the spinor takes the form

$$L_I = ig \sum_{i=1}^{m} C^\mu(X)A_\mu(Y - \tfrac{1}{2}\epsilon_i) + \sum_{i=1}^{m} C^\mu(X)A_\mu(Y + \tfrac{1}{2}\epsilon_i)$$
$$= ig \sum_{i=1}^{n} \int d^4r\, d^4s\, d^4t [\bar{\psi}(X, r)r^\mu \psi(X, S)A_\mu(Y, t)$$
$$+ \bar{\psi}(X, r)r^\mu \psi(X, s)A_\mu(\bar{Y}_i, t)] \tag{2.12}$$

where $Y_i = Y - \frac{1}{2}\epsilon_i$, $\bar{Y}_i = Y + \frac{1}{2}\epsilon_i$, and g is the dimensionless weak coupling constant, which is taken to have the value $g = 10^{-10}\, e$ (Bandyopadhyay, 1968).

Now taking m such that $e/2m = g$, the weak coupling constant, we note that the system of interactions (2.12) in the limit $\epsilon \to 0$ just reduces to the formal electromagnetic coupling

$$ie \int d^4r\, d^4s\, d^4t \bar{\psi}(X, r)r^\mu \psi(X, s)A_\mu(Y, t) \tag{2.13}$$

Thus in the limit $\epsilon \to 0$, n photon weak interactions can be considered to be "equivalent" to the proper electromagnetic interaction (2.13), and by this a geometrical description of e in terms of g is obtained.

To show that the coupling constant e obtained in such a manner actually represents the "charge," we have shown in I that the equation (2.13) can give rise to a less symmetric solution that generates the "electromagnetic" interaction from a system of n photon "weak" interactions at different space-time points. This violation of symmetry occurs owing to the fact that (1) the interaction involving nonlocal fields such as equation (2.13) is equivalent to an interaction involving local fields with form factor which introduces a cut-off giving mass to the bare spinor and (2) the positive and negative sign of the form factor is found to correspond to the positive and negative energy states and thus a four-component spinor can be formed from a bare two-component spinor. The generation of mass through the interaction violates the summetry corresponding to the invariance under the transformation $\psi \to e^{i\alpha\gamma_5}\psi$ inherent in the original system of "weak" interactions involving the bare two-component spinor as given by equation (2.12), and thus, in this scheme, electromagnetic interaction is generated through the spontaneous breakdown of symmetry and the photon appears as a Goldstone boson.

It is to be noted here that the number n of the system of weak interactions in equation (2.12) bears a very crucial sense: n must be a unique number, otherwise we could get any amount of charge and mass of a lepton formed in this manner. Since we know that all charges in nature occur in units of e (provided we assume that there are no fractionally charged particles like quarks), n must be specified by the quantity e/g, where g is the photon-neutrino weak coupling constant. Again in Bandyopadhyay (1974) we have argued that this number n can be specified if we assume that space-time in nature is quantized such that the whole "space-time continuum" is considered as a collection of "elementary space-time domains." Each such domain is specified by the fact that no physical measurement is possible within it. This elementary domain can be considered to be the "seat" of an elementary particle like an electron or a muon. The number n of interactions as specified by the ratio e/g is determined by the dimension of this domain such that no more interacting photons in the system of weak interactions at different space-time points with an extended structure of a neutrino as given by equation (2.12) can be accommodated within this quantized domain. Thus we can get the unique charge e for a lepton. This concept of space-time quantization requires the introduction of a fundamental length l_0 in nature as a measure of the linear dimension of the elementary domain.

From the above analysis, we now show that if we assume that the internal space within the quantized domain is not governed by Lorentz symmetry, we do not face any difficulty as to the relativistic invariance of the form factor involved in the electromagnetic interaction of a charged lepton. To this end, let us first consider a single photon–neutrino weak interaction where both photons and neutrinos are taken to have extended structures. In nonlocal field theory, the interaction can be depicted as

$$L_I = ig \int d^4r\, d^4s\, d^4t \bar{\psi}(X, r)\gamma_\mu \psi(X, s) A_\mu(Y, t) \tag{2.14}$$

where X and Y are external variables and r, s, t are internal variables. When this interaction is transformed into the nonlocal interaction theory involving local fields with a form factor, the interaction (2.14) reduces to the form

$$L_I = ig \int d^4X_1\, d^4X_2\, d^4X_3 \bar{\psi}(X_1) r_\mu \bar{\psi}(X_2) F(X_1, X_2, X_3) A_\mu(X_3) \tag{2.15}$$

where the form factor $F(X_1, X_2, X_3)$ is related to the internal space variables through a relation of the form

$$F(X_1, X_2, X_3) = g\bar{X}(X_1 - X_3)\, \delta \left[\frac{(X_1 + X_3)}{2} - X_2 \right] \tag{2.16}$$

From our above analysis, we note that the form factor depends on the complex conjugate of the internal eigenfunction and the internal space variable is here characterized by the relation $r < l_0$. *So, if Lorentz invariance is taken to be violated within the region characterized by the domain* $r < l_0$, the form factor $F(X_1, X_2, X_3)$ will also not observe relativistic invariance. However, since no measurement is possible within the quantized domain in the physical world, it does not matter if the form factor $F(X_1, X_2, X_3)$ violates relativistic invariance. Also we observe that although the massless particles like photons and neutrinos are taken to be of extended structure, the dimension of these extended particles will not be *observable* since the linear dimension of these particles will be less than l_0. Thus we see that although the massless particles like photons and neutrinos are taken to be of extended structures in the physical world they will only behave as point particles. In fact, only when the quantized domain is filled up by n interacting photons does the dimension of the region covering all these interactions become $l = l_0$. In this case, the interaction (2.13) involving nonlocal fields, when transformed into the non-local interaction theory involving form factor, reduces to the form

$$L_r = ie \int d^4X_1\, d^4X_2\, d^4X_3\, \bar{\psi}(X_1)\gamma_\mu \psi(X_2) F(X_1 - X_3, X_2 - X_3) A_\mu(X_3) \quad (2.17)$$

and the form factor in equation (2.17) depends on the internal eigenfunctions through a relation of the form

$$F = g\bar{X}(X_1 - X_3)\, \delta\, [(X_1 + X_3)/2 - X_2]$$

and the linear dimensions of the internal space here becomes identical with l_0. Thus the relativistic invariance for the form factor $F(X_1, X_2, X_3)$ is fully maintained when the quantized domain becomes the seat of an electron (or a muon).

Now we make some remarks about Yukawa's notion that the internal quantum numbers of hadrons can be related with the degrees of freedom associated with the internal space of the extended structure of hadrons. In fact, if this is the situation, then we are not justified in assuming that the internal space is not governed by the Lorentz group. However, it is now almost certain that hadrons are all composite particles. In a recent paper (Bandyopadhyay, 1975) we have shown that the internal quantum numbers like isospin, strangeness, and baryon number can be interpreted from the very configuration of hadrons when hadrons are taken to be composed of three spin-$\frac{1}{2}$ particles (which may be identified with μ^+, γ_μ, and μ^- respectively) and the internal motion is quantized in units of $\frac{1}{2}\hbar$ instead of $\hbar$ in such a way that the third components $+\frac{1}{2}$ and $-\frac{1}{2}$ represent particles and antiparticles or vice versa. Thus if we take this picture of hadrons, it is not necessary to take into account the internal degrees of freedom of an extended particle to account for the internal quantum number.

3. *Nonlocal Field Theory and the Problem of Convergence*

In this section, we would like to point out that the concept of space-time quantization helps us to ensure convergence in nonlocal interaction theories. In fact, the nonlocal interaction theory was first proposed to avoid the divergence difficulties encountered in local field theory. However, Bloch (1952), Kristensen and Moeller (1952), Moeller (1953), and Pierles (1953) have shown that to ensure convergence we must impose certain constraints on the nature of the form factor and this may lead to the violation of causality. In fact, as Moller (1953) has shown, to obtain a convergent expression for the matrix element of the field variables $\psi(x)$, $\langle a|\psi(x)|b\rangle$ it is necessary to assume that the factor $g(l_1, l_3) = 0$ for $l_1 > 0$ (space-like g) where $g = p_b - p_a$, $l_3 = p - p^b$ (Bloch's condition). Since on account of the reality requirement $g(l_1, l_3) = g^*(-l_3, -l_1)$, g must also be zero for space-like l_3. But, as is evident, a form factor satisfying Bloch's condition is not reconcilable with the principle of correspondence and causality in the large. The principle of correspondence suggests that if the field functions only contain waves of wavelength large compared with a certain constant λ of the dimension of a length, the form factor is effectively a δ function like in the local field theory. That is, the form factor depends on the constant λ in such a way that $F(\lambda, x_1, x_2, x_3)$ in the limit $\lambda \to 0$, goes over into the corresponding quantity of the local theory. That is, in the momentum space, it would read

$$\lim_{\lambda \to 0} g(l_1, l_3) = 1$$

But the Bloch's condition for convergence requires that

$$\begin{aligned} \lim_{\lambda \to 0} g(l_1, l_3) &= 1 \qquad \text{for time-like } l_1, l_3, l_1 + l_3 \\ &= 0 \qquad \text{for space-like } l_1, l_3, l_1 + l_3 \end{aligned}$$

This would also probably imply the violation of the "causality condition in the large" which states that if Ω and Ω' are two clearly separated domains in space-time, whose linear dimensions are large compared with λ, then any signal transmitted from Ω to Ω' should take place with velocity smaller than c, and further the absorption process should occur later than the emission process.

From our present analysis, we here note that the principle of correspondence has a different meaning if the space-time is quantized. For then the limit $\lambda \to 0$ becomes meaningless as l_0 is the minimum linear dimensions we can achieve. In this picture, we find that if the field functions only contain waves of wavelength very large compared with l_0, the interaction will "appear" as a local one and the form factor can "approximately" be taken to be unity. Moreover, Bloch's conditions are automatically satisfied for quantized space-time, because then we can define a unit time-like vector N_μ and any energy-momentum vector p_μ can be replaced by the universal vector N_μ. It may be noted that the introduction of this vector N_μ is equivalent to going back to the absolute space and time. But we do not face any contradiction since our space-time is quantized, and within a quantized space-time domain we have already abandoned the requirement of Lorentz invariance.

4. *Discussion*

From the above arguments, we find that the main objections that were raised for the violation of Lorentz symmetry of the internal space can be avoided when the concepts of space-time quantization and dynamical origin of charge and mass of an electron are introduced. *Also, as mentioned above*, the introduction of Lorentz noninvariance in the internal space helps us to avoid the infinite degeneracies associated with nonlocal fields for extended particles. Again, we find that massless particles like photons and neutrinos, although they may be taken to be of extended structure, in the physical world will only appear as point particles since the dimension of these particles will be less than that of the quantized domain.

Finally, it may be mentioned that Tati (1960) and Blokhintzev (1964) also introduced a unit time-like vector N_μ assuming that the space-time world as a whole has a preferred direction due to the distribution of matter and energy and in this way appealed to Mach's principle. *In the present case, although we have assumed that Lorentz symmetry is violated within the quantized domain*, this comes into being when the primary unit of charge and mass is formed and the elementary domain becomes the seat of an electron (or muon). Thus in our present picture, Lorentz symmetry may be considered to be a consequence of the distribution of matter and energy in the universe. This may be taken to be another interpretation of the famous Mach principle.

References

Bandyopadhyay, P. (1968). *Physical Review*, **173**, 1481.
Bandyopadhyay, P. (1973a). *International Journal of Theoretical Physics*, 8, 323.
Bandyopadhyay, P. (1973b). *Progress of Theoretical Physics*, **30**, 563.
Bandyopadhyay, P. (1974). *International Journal of Theoretical Physics*, **10**, 175.
Bandyopadhyay, P. (1975). *Particles and Nuclei* (to be published).
Bloch, C. (1952). *Kongelige Danske Videnskabernes Selskab, Matematisk-Fysiske Meddelelser*, **27**, No. 8.
Blokhintzev, D. I. (1964). *Physics Letters*, **12**, 272.
Kristensen, P., and Moller, C. (1952). *Kongelige Danske Videnskabernes Selskab, Matematisk-Fysiske Meddelelser*, **27**, No. 7.
Moller, C. (1953). Proceedings of the International Conference of Theoretical Physics (Kyoto and Tokyo), p. 13.
Pais, A. (1953). Proceedings of the International Conference of Theoretical Physics (Kyoto and Tokyo), p. 7.
Pierles, R. E. (1953). Proceedings of the International Conference of Theoretical Physics (Kyoto and Tokyo), p. 24.
Tati, T. (1960). *Progress of Theoretical Physics*, **24**, 1.
Yukawa, H. (1950). *Physical Review*, **77**, 219.
Yukawa, H. (1965). Proceedings of the International Conference on Elementary Particles, p. 139.
Yukawa, H. (1973). Proceedings of the International Conference of Theoretical Physics (Kyoto and Tokyo), p. 2.